Chemistry 410A **Exam 1** **Spring 2008**

Solutions

1. Two classical beams of radiation obey the equation

$$\mathcal{E}_{\rm A} = \epsilon_{\rm A} \sin(2\pi\nu_{\rm A} t), \qquad \mathcal{E}_{\rm B} = \epsilon_{\rm B} \sin(2\pi\nu_{\rm B} t).$$

The two beams overlap starting at a **position** $x = 0$, where both fields are exactly zero and about to become positive. Find the first x value (in m) **greater** than zero where the electric field is again exactly zero if $\nu_{\rm A} = 1.0 \cdot 10^{14}$ s^{-1} and $\nu_{\rm B} = 1.2 \cdot 10^{14}$ s^{-1}. **Solution:** The combined wave will be zero the next time that the two waves are both zero, which occurs at intervals of $\lambda/2$. We convert the time-dependent functions to x-dependent functions by setting $x = ct$ (distance is speed time time):

$$\mathcal{E}_{\rm A} = \epsilon_{\rm A} \sin(2\pi\nu_{\rm A} x/c) = \epsilon_{\rm A} \sin(2\pi x/\lambda_{\rm A})$$
$$\mathcal{E}_{\rm B} = \epsilon_{\rm B} \sin(2\pi x/\lambda_{\rm B})$$
$$\frac{\lambda_{\rm A}}{\lambda_{\rm B}} = \frac{c/\nu_{\rm A}}{c/\nu_{\rm B}} = \frac{1.2}{1.0} = \frac{6}{5}$$

Both functions will be zero at the same time at a distance $5\lambda_{\rm A}/2 = 6\lambda_{\rm B}/2$r:

$$\frac{5\lambda_{\rm A}}{2} = \frac{5c}{2\nu_{\rm A}} = \boxed{7.5 \cdot 10^{-6}\ \rm m}$$

2. Li^{2+} absorbs a photon with energy 6.0 $E_{\rm h}$, ionizing the electron. The remaining energy goes into the kinetic energy of the ionized electron. Calculate its de Broglie wavelength. **Solution:** The ionization energy of Li^{2+} is $(Z^2/2)\ E_{\rm h} = 4.5\ E_{\rm h}$, so the excess energy is 1.5 $E_{\rm h}$. Next, we convert that to a kinetic energy in J and solve for the momentum to get $\lambda_{\rm dB}$:

$$K = 1.5\ E_{\rm h} = 6.54 \cdot 10^{-18}\ {\rm J} = \frac{m_e v^2}{2} = \frac{p^2}{2m_e}$$
$$\lambda_{\rm dB} = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$
$$= \frac{6.626 \cdot 10^{-34}\ {\rm J\ s}}{[2(9.109 \cdot 10^{-31}\ {\rm kg})(6.54 \cdot 10^{-18}\ {\rm J})]^{1/2}}$$
$$= 1.92 \cdot 10^{-10}\ {\rm m} = \boxed{1.92\ \text{Å}}.$$

3. Calculate the **minimum** time (in seconds) necessary for the electron to change from the $n = 1$ state to the $n = 2$ state in the Bohr model, assuming that the electron travels at the speed $v_{n=1}$ during the transition. **Solution:** The shortest distance between the two orbits is $r_2 - r_1$, where $r_n = (n^2/Z)a_0$, and dividing this by the

speed v_1 gives us the minimum possible time for the transition. For hydrogen, $Z = 1$.

$$t \approx \frac{r_2 - r_1}{v_1} = \frac{[(2^2/Z) - (1^2/Z)]a_0}{Ze^2/(4\pi\epsilon_0(1)\hbar)}$$
$$= \frac{3a_0(4\pi\epsilon_0)\hbar}{e^2} = \frac{3(5.292 \cdot 10^{-11}\text{ m})(1.113 \cdot 10^{-10}\text{ C}^2\text{ J}^{-1}\text{ m}^{-1})(1.055 \cdot 10^{-34}\text{ J s})}{(1.602 \cdot 10^{-19}\text{ C})^2}$$
$$= \boxed{7.3 \cdot 10^{-17}\text{ s}}$$

4. We have a wavefunction $\psi(x) = 2xe^{-3x^2}$. Circle the letter for any operator where $\psi(x)$ is an eigenfunction of the operator, and write the eigenvalue.

   (a) $4x$ $\boxed{\text{No}}$ because we get a function of the form $x^2e^{-3x^2}$.

   (b) $(1/x)\frac{d}{dx}$ $\boxed{\text{No}}$ because we get a combination of functions with different powers of x

   (c) $\frac{d}{dx}(1/x)$ $\boxed{\text{Yes}}$:

$$\frac{d}{dx}\frac{1}{x}\psi(x) = \frac{d}{dx}(2e^{-3x^2}) = -12xe^{-3x} = \boxed{-6}\psi(x)$$

   (d) $x^3e^{-3x^2}\frac{d}{dx}(e^{3x^2}/x^2)$ $\boxed{\text{Yes}}$:

$$x^3e^{-3x^2}\frac{d}{dx}\frac{e^{3x^2}}{x^2}\psi(x) = x^3e^{-3x^2}\frac{d}{dx}\left(\frac{2}{x}\right) = -2xe^{-3x^2} = \boxed{(-1)}\psi(x)$$

5. Begin with a particle of mass m_e and charge $-e$ in the $n = 1$ state of a one-dimensional box of length a. Define ΔE_{12} to be the $n = 1 \rightarrow 2$ transition energy. For each change listed in the table below, indicate the factor by which the new transition energy ΔE changes, compared to this initial value ΔE_{12}. **Solution:** We're using the energy equation $E_n = \pi^2 n^2\hbar^2/(2ma^2)$, which gives

$$\Delta E = \frac{\pi^2\hbar^2}{2ma^2}\left(n'^2 - n''^2\right)$$

change initial system by	$\Delta E = \Delta E_{12}$ **times** ...
increasing mass to m_p (the proton mass)	m_e/m_p
reducing box length to $a/2$	4
turning box to point along z axis instead of x axis	1 (no change)
increasing upper state of transition to $n = 4$	$n'^2 - n''^2$ changes from 3 to 15 so **5**
increasing charge to $-2e$	1 (no change)