Problem Set 1 Solutions

1. **Problem 2.5.** The maximum probability is the maximum of $|\psi|^2 = (2/l) \sin^2(n\pi x/l)$, and occurs anywhere that the wavefunction reaches its greatest positive or negative amplitude. Any sine wave has a maximum at $(4m + 1)\pi/2$ and a minimum at $(4m + 3)\pi/4$, and is zero at $m\pi$, where $m$ is a non-negative integer. For our sine waves, therefore, the probability density reaches its maxima at $n\pi x/l = (4m + 1)\pi/2$ or $(4m + 3)\pi/4$, and minima (where the probability density is zero) at $n\pi x/l = m\pi$. For $n = 1$, the probability density reaches a maximum at $\pi x/l = \pi/2$, which corresponds to $x = l/2$, and a minimum at $\pi x/l = 0$ or $\pi$, which corresponds to $x = 0$ or $x = l$. Similarly, for $n = 2$ the maximum in the probability density is at $2\pi x/l = \pi/2$, or $x = l/4$, and at $2\pi x/l = 3\pi/2$, or $x = 3l/4$.

2. **Problem 2.6.** The probability of being in the left quarter of the box is the integral of the probability density $|\psi|^2$ over that range, and is given by

$$
\int_0^{l/4} |\psi(x)|^2 \, dx = \frac{2}{l} \int_0^{l/4} \sin^2 \left( \frac{n\pi x}{l} \right) \, dx
$$

$$
= \frac{2}{l} \left[ \frac{l}{8} - \frac{l\sin(n\pi/2)}{4n\pi} \right]
$$

$$
= \frac{1}{4} \frac{\sin(n\pi/2)}{2n\pi}.
$$

I used the general solution to the integral of $\sin^2(ax)$ in solving this:

$$
\int \sin^2(ax) \, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} + C.
$$

The answer is an oscillating function with a maximum at $n = 3$ (which is easiest to find out by trial and error) and it quickly converges towards 1/4 as $n$ increases.

3. **Problem 2.9.** The lowest two energy levels are $n = 1$ and $n = 2$, and the transition energy is

$$
\Delta E = \frac{\hbar^2}{8ml^2} (2^2 - 1^2) = \frac{3\hbar^2}{8ml^2} = 1.81 \cdot 10^{-17} \text{ J}.
$$

For the final value, I plugged in $m_e = 9.109 \cdot 10^{-31}$ kg for the mass and $l = 1.0$ Å. We use Planck’s law to get the wavelength:

$$
\lambda = \frac{hc}{\Delta E} = 1.10 \cdot 10^{-8} \text{ m} = 11.1 \text{ nm}.
$$

This is in the UV.

4. **Problem 2.11.** This is an algebra problem, where we take the expression for $\Delta E$ in a 1D box and solve for $l$:

$$
\Delta E = \frac{\hbar^2}{8ml^2} (5^2 - 2^2) = \frac{21\hbar^2}{8ml^2} = h\nu
$$

$$
l = \left( \frac{21\hbar}{8m\nu} \right)^{1/2} = 1.78 \cdot 10^{-9} \text{ m} = 1.8 \text{ nm}.
$$