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Experiment 1

Exponential Notation

Introduction

Scientific study includes measurements and quantities that are tremendously large and some that are inconceivably small. For example, there are approximately 3,350,000,000,000,000,000,000 molecules of sucrose in every teaspoon of table sugar. This is not surprising if one considers that the distance between carbon atoms in a molecule of sucrose is 0.000 000 0015 meters.

Both of these numbers are more easily written using scientific notation, 3.35×10^{21} and 1.5×10^{-9}, respectively. The exponential format is of this form, $A \times 10^n$, where A is a digit between 1 and 10 and the exponent n is a positive or negative integer. Not only does scientific notation make very large and very small numbers more convenient to write, but it also allows for the recording of measurements and calculations without ambiguous zeroes.

Powers of 10

A power of 10 is the result of 10 raised to some exponential value. For example, "ten squared" is written 10^2. This means that the number 10 is multiplied by itself or 10×10. Ten raised to the third power is 10^3 or $10 \times 10 \times 10$. Ten raised to the nth power is 10^n or $10 \times 10 \times 10 \ldots n$ (this is 10 multiplied by itself n times).

Since $10^2 = 100$ and $10^3 = 1000$, we can see that the number of zeros is equal to n. Thus, $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$. One million is written in decimal form as 1,000,000 which is one, followed by six zeros; in scientific notation, one million is written as 10^6.

So far, our examples have been numbers that are greater than ten, 100, 1000, 100,000 and a million. Numbers that are *less than one* can also be expressed with powers of ten; the difference is that the *exponent will be negative.*

Converting Ordinary Decimal Numbers into Scientific Notation

If a number is ten or greater, we move the decimal point to the left by n places to obtain a number between one and ten. That result is then multiplied by a power of 10; actually it is multiplied by ten raised to the n^{th} power. Here is an example, the approximate distance between the earth and the sun is 93 million miles or 93,000,000 mi. In scientific notation we write 9.3×10^7 mi.

decimal point is moved to obtain a number between 1 and 10

$$93000000. = 9.3 \times 10^7$$

decimal point moved seven places to the left

If a number is less than one, the decimal point is shifted to the right until, again, we obtain a number between one and ten. Then multiply that number by 10^{-n} where n is equal to the number of places that we moved the decimal point. Here is an example. The size of bacteria is about 0.0000322 inches. In scientific notation we write, 3.22×10^{-5} in.

decimal point is moved to obtain a number between 1 and 10

$$0.0000322 = 3.22 \times 10^{-5}$$

decimal point moved five places to the right

Look at some more examples:

450,000 lbs. = 4.5×10^5 lbs.

385.26 h = 3.8526×10^2 h

0.03307 cm = 3.307×10^{-2} cm

0.0005 L = 5×10^{-4} L

Again notice that numbers greater than 10 have positive exponents and numbers less than 1 have negative exponents.

Finally, consider this last example: we usually write 87,200 as 8.72×10^4 but we could also express this same number as 87.2×10^3 or 872×10^2. The only difference is the number of places that we move the decimal point. However, the standard form of scientific notation uses a number between 1 and 10; thus $8.72 \text{ X } 10^4$ is the usual expression for this number.

There is one last point to consider when writing any number that is part of a measurement, whether the number is in decimal form or in scientific notation. Every number must include the unit of measurement such as, inches (in), meters (m), milliliters (mL), seconds (s), etc.

Converting Scientific Notation into Decimals

If a number is written in scientific notation and we wish to convert it into a decimal number, we simply reverse the process described above. For example, 5.92×10^5 kg can be written in decimal form as 592,000 kg. To make this conversion, we drop the power of 10 and move the decimal point five places to the right. Remember, a positive exponent means we are dealing with a number greater than 10 so in order to make 5.92 a bigger number we must add zeros which is done by moving the decimal point to the right. Recall

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that we moved the decimal to the left to express 592 thousand in scientific notation, and the reverse process requires that we move the decimal back to the right.

$$592{,}000. = 5.92 \times 10^5$$

decimal point is moved to the left to convert this decimal number into scientific notation

$$5.92 \times 10^5 = 592{,}000.$$

decimal point is moved to the right to convert this notation into a decimal

The number 9.7×10^{-2} kg is written in decimal form as 0.097 kg. Again, the power of 10 is dropped and the decimal point is moved 2 places to the left.

$$0.097 = 9.7 \times 10^{-2}$$

decimal point is moved to the right to convert this decimal number into scientific notation

$$9.7 \times 10^{-2} = 0.097$$

decimal point is moved to the left to convert this notation into a decimal

Adding and Subtracting Exponential Numbers

To add or subtract exponential numbers, both numbers must have the same exponent. For example, in this problem, the exponents are not the same: $2.3 \times 10^2 + 1.4 \times 10^3$. One of the numbers must have its exponent changed:

exponent changed (decimal point moved to the left)

$$\begin{array}{rl} & 0.23 \times 10^3 \\ + & 1.4 \times 10^3 \\ \hline & 1.63 \times 10^3 \end{array}$$

don't Worry about This- you will always be able To use a Calculator!

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Using a Calculator for Scientific Notation

To use exponential notation with a calculator, you must have an exponent key. This key is usually labeled as EE or EXP. To enter this number, 6.02×10^{23}, use the following sequence of keystrokes:

$\boxed{6}\ \boxed{\cdot}\ \boxed{0}\ \boxed{2}\ \boxed{\text{EE}}\ \boxed{2}\ \boxed{3}$

The calculator display may look like this:

```
6.02 23
```

To enter a negative exponent, use the +/- key before the exponent value; 7×10^{-3} is entered:

$\boxed{7}\ \boxed{\text{EE}}\ \boxed{+/-}\ \boxed{3}$

The display may look like this:

```
7      -03
```

Notice that you do not enter × (multiplication key) or 10; that is what the EE key does for you.

When adding, subtracting, multiplying or dividing exponential numbers with a calculator, the numbers do not have to have the same exponent; the calculator reconciles this automatically.

Here are two more examples illustrating the keystrokes and the displayed result:

$$(6.2 \times 10^{-9})\ (5.7 \times 10^{12}) = 3.534 \times 10^{4}$$

$\boxed{6}\ \boxed{\cdot}\ \boxed{2}\ \boxed{\text{EE}}\ \boxed{+/-}\ \boxed{9}$

$\boxed{\times}$

$\boxed{5}\ \boxed{\cdot}\ \boxed{7}\ \boxed{\text{EE}}\ \boxed{1}\ \boxed{2}$

The display may look like this:

```
3.534   04
```

Here is another example,

$$(5.2 \times 10^{-3}) + (1.4 \times 10^{-2}) = 1.92 \times 10^{-2}$$

$\boxed{5}\ \boxed{\cdot}\ \boxed{2}\ \boxed{\text{EE}}\ \boxed{+/-}\ \boxed{3}$

[+]

[1] [·] [4] [EE] [+/-] [2]

The display may look like this:

1.92 -02

Using a Calculator to Find Square Roots

Locate the square root key, $\sqrt{x}$. To find the square root of any number, most calculators require you to enter the number followed by the square root key. Here are some examples (some digits from the calculator have been dropped):

$$\sqrt{31.7} = 5.6302\ldots$$

The key sequence is:

[3] [1] [·] [7] [√]

The display will look like:

5.6302...

$$\sqrt{5.6 \times 10^5} = 748.331\ldots$$

The key sequence is:

[5] [·] [6] [EE] [5] [√]

The display will look like:

748.331...

Using a Calculator to Find Higher Powers of Decimal Numbers

Oftentimes we need to have a decimal number raised to some power; for example we frequently need to square a number. For this purpose, use the x^2 key. For example:

$$45^2 = 45 \times 45 = 2025$$

The key sequence is:

[4] [5] [x^2]

The display will look like:

2025

If we need to cube a number or find a higher power we use this key, y^x.

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Here are two examples:

$71^3 = 357{,}911$

The key sequence is:

$\boxed{7}$ $\boxed{1}$ $\boxed{y^x}$ $\boxed{3}$

The display will show:

```
357911
```

Here is another example,

$(12.6)^5 = 317{,}579.6938$

The key sequence is:

$\boxed{1}$ $\boxed{2}$ $\boxed{\cdot}$ $\boxed{6}$ $\boxed{y^x}$ $\boxed{5}$

The display will show:

```
317,579.6938
```

Logarithms and Inverse Logarithms

Logarithms

Any positive number x can be written as 10 raised to some power z. The format is: $x = 10^z$.

The exponent z is the common log of x; sometimes z is called the base 10 log of x. We write expressions such as: $\log_{10} x = z$ or just $\log x = z$. Again, here are the mathematical relationships:

$x = 10^z$ $\qquad\qquad$ $\log x = z$

Think of a log as the exponent; a definition of a log is the power z to which 10 is raised so that it equals x. For example, 100 can be expressed as 10^2 and therefore, the log of 100 is 2:

since $100 = 10^2$ $\qquad\qquad$ the $\log 100 = 2$

Here are more examples:

$1 = 10^0$	therefore $\log 10 = 1$
$10 = 10^1$	therefore $\log 100 = 2$
$100 = 10^2$	therefore $\log 0.01 = -2$
$0.01 = 10^{-2}$	therefore $\log 1 = 0$

Notice that the log of 1 is zero, the log of a number greater than 1 is positive and the log of a number less than 1 is negative. The log of a negative number is undefined because 10 raised to any power is always positive.

Taking the log of powers of 10 is straightforward, we just read the exponent on 10. How do we deal with other decimal numbers and exponential numbers? We use the log key on the calculator. Look at the following examples and the key sequence required for each:

$\log 4.5 = 6.532$ … (some digits have been dropped)

The key sequence is:

$\boxed{4}$ $\boxed{\cdot}$ $\boxed{5}$ $\boxed{\log}$

The display will show:

~~6.532...~~ 0.6532

Try entering log 8.7×10^3 = 3.9395...

The key sequence is:

$\boxed{8}$ $\boxed{\cdot}$ $\boxed{7}$ $\boxed{EE}$ $\boxed{3}$ $\boxed{\log}$

The display will show:

3.9395...

Enter, log $4.1 \times 10^{-2} = -1.3872...$

The key sequence is:

$\boxed{4}$ $\boxed{\cdot}$ $\boxed{1}$ $\boxed{EE}$ $\boxed{+/-}$ $\boxed{2}$ $\boxed{\log}$

The display will show:

-1.3872...

Antilogarithms

Inverse logs, sometimes called antilogs, are obtained by the reverse process: If z is the log of x, then x is the antilog of z.

$$\text{if} \quad z = \log x \qquad \text{then } x = \text{antilog } z$$

For example, the antilog of 3 is 10^3 which is 1000. The antilog of 0.25 is $10^{0.25}$ which is 1.778. To find the antilog of 3.2, on your calculator, you can use either the y^x key or 10^x key. The 10^x key is usually the 2nd function above the LOG key. The next two examples illustrate both methods for finding the antilog of a number.

Using the 10^x key:

Find the antilog of 3.2

This is how to activate the 10^x key

$\boxed{3}$ $\boxed{\cdot}$ $\boxed{2}$ $\underbrace{\boxed{INV}\ \boxed{LOG}}$

This gives the 2nd function for the LOG key which is the 10^x key

The display will show:

1584.89...

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Using the y^x key:

Find the antilog of 3.2

$\boxed{1}\ \boxed{0}\ \boxed{y^x}\ \boxed{3}\ \boxed{\cdot}\ \boxed{2}\ \boxed{=}$

The display will show:

1584.89...

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SIGNIFICANT FIGURES

Introduction

Significant figures are the number of digits used in a measurement. Suppose your class assignment was to find the mass of a 1-cent coin, a penny. There are several types of balances in the room and no further instructions are given as to which balance to use. Each student weighs the same coin on a similar balance and records their measurement on a class data sheet. The values recorded by the students are listed in Table 2.1.

Table 2.1

2.900 g	2.844 g
2.860 g	2.843 g
2.850 g	2.890 g

At this point it should be obvious that any measurement involves a certain degree of uncertainty. **Uncertainty** is an indication of how confident we are that our number represents the actual value. What is the actual value? Scientists often state that the actual value can never be known. All we can do is measure a value (mass in this case) and interpret the result as having some degree of uncertainty.

Accuracy and Precision

When we discuss uncertainty, we use the terms **accuracy** and **precision**. In common usage, we use these terms interchangeably but scientists have strict definitions for each. **Accuracy** refers to how well our experimental value compares to the actual value. The only instance where accuracy can be calculated is when the actual value has been previously determined. For example, the density of mercury is 13.6 g/mL. If a student measures the density of mercury and obtains a value of 14.5 g/mL, the accuracy can be calculated. This calculation would be expressed as a **percent error**:

$$\%\ \text{error} = \left|\frac{\text{actual value} - \text{experimental value}}{\text{actual value}}\right| \times 100\ \%$$

$$\%\ \text{error} = \left|\frac{13.6 - 14.5}{13.6}\right| \times 100 \qquad \text{(EQ 2.2)}$$

$$\%\ \text{error} = 6.62\ \%$$

Precision refers to how well a number of independent measurements agree with one another. For example, if we measured the mass three times and got numbers that varied

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Rules for Determining the Number of Significant Figures

1. All non-zero digits are significant.
2. Zeros in the middle of a number are significant; 402 has three significant figures.
3. Zeros at the beginning of a number are not significant; these zeros only locate the decimal point. For example, 0.0034 has only two significant figures.
4. Zeros at the end of a number that come after the decimal point are significant. For example, 69.430 g has five significant figures. If the last digit was not significant, it would not have been recorded. This number tells the reader that the measurement was made to the nearest 1/1000th of a gram. It just so happens that the last digit on the balance was zero, but it could have been a 2 or a 7, for example.
5. Zeros at the end of a number that come before the decimal point may or may not be significant. 24,000 L may have 2, 3, 4, or 5 significant figures. We do not know if the zeros are being used to hold the decimal point or if the zeros are part of the measurement. Numbers of this type are the most difficult to deal with since not everyone agrees on how to handle the zeros before a decimal point. Fortunately this ambiguity can be avoided if numbers are expressed in scientific notation. For example, if the number 24,000 L is written as 2.4×10^4 L, we know there are only two significant figures. If the volume measurement had been made to the nearest milliliter, then all digits would be significant and it would be expressed as 2.4000×10^4 L.

Using Significant Figures in Calculations

Rounding Off Numbers in Addition and Subtraction

When numbers are added or subtracted together, the uncertainty of the result depends on the precision of the least precise number used in the calculation. The least precise number is the number with the fewest **decimal places**. Because of this rule, the calculated result is rounded off and some digits may be dropped.

For example, if we weighed two coins with a mass of 2.356 g and 2.2 g and then added the masses, we obtain 4.556 g. It should be obvious that the second coin was weighed on a balance with a precision of 1/10th of a gram. It makes no sense to express the final result (4.556 g) to the nearest 1/1000th of a gram since one of the measurements was made with a precision much less than 1/1000th of a gram. In other words, the final result is limited by the least precise measurement, 2.2 g, and therefore, only one digit past the decimal is allowed in the final answer and the remaining digits must be dropped. When we drop the extra digits, the last remaining digit is left unchanged or rounded up by one depending on the value of the dropped digits.

Rules for rounding. If the dropped digit is 5 or greater, the last remaining digit is increased by one; here is an example:

2.356 g + 2.2 g = 4.556 g is rounded to one decimal place and the digit is rounded up to give 4.6 g

10.432 g + 9.01 g = 19.442 g rounded to two decimal places to give 19.44 g

6.705 g – 2.1 g = 4.605 g rounded to one decimal place to give 4.6 g

536.12 g – 365.994 g = 170.126 g rounded to two decimal places to give 170.13

12.444 + 32.1 + 678.23 + 65 = 787.774 rounded to the nearest ones place to give 788 greatly, we would say that our precision was poor. Precision also refers to the manner in which the measurement was obtained. A particular measurement is constrained to the precision of the measuring device or technique. For example, if we use a single-pan balance that measures to the nearest tenth of a gram, our precision is ± 0.1 g; we would say that our measurement is precise to within one tenth of a gram. A value of 2.9 g indicates that the actual mass lies within the range of 2.8 to 3.0 grams. In this case, we record the mass as:

$$2.9 \pm 0.1 \text{ g} \qquad \text{(EQ 2.3)}$$

If we use an electronic balance that measures to the nearest 1000th of a gram (as that data shown in Table 2.1), then our precision increases to ± 0.001 g and we record the mass as:

$$2.844 \pm 0.001 \text{ g} \qquad \text{(EQ 2.4)}$$

It is understood that the last digit in any measurement is the estimated digit. Sometimes the last digit is called the "doubtful digit" or "unreliable digit".

Scientists communicate the level of uncertainty by the use of significant figures. A mass of 2.9 g contains two significant figures; a mass of 2.8439 g contains 5 significant figures. We can see that the value of 2.8439 g is more precise than 2.9 g, because the greater the number of significant figures, the greater the precision of the measurement. Proper use of significant figures eliminates the need to tell the reader that a single-pan balance was used or an electronic balance was used. Again, it is understood that the last digit is the estimated digit.

One final point about numbers should be mentioned. Scientists refer to numbers as exact or inexact. Exact numbers are numbers that we obtain by counting small groups of objects or numbers obtained by definition. For example, we can count the fingers on our hand and get an exact number (most people have 5). There is no uncertainty in this result, but we cannot count large groups of objects without some degree of uncertainty. For example, the number of stars in our galaxy is not an exact number. Other kinds of exact numbers are the result of mathematical definitions or conversion factors. For example, one kilogram is equal to exactly 1000 grams.

Inexact numbers are those that we have already discussed; all measurements are inexact numbers because they are estimates of the actual value and contain some degree of uncertainty. Some conversion factors can be expressed to varying degrees of precision as well. When we convert English units to the corresponding metric unit, the conversion is not always exact. For example, one English pound expressed in metric units of grams is:

$$1 \text{ lb.} = 453.59 \text{ g} \qquad \text{(EQ 2.5)}$$

We could also use a less precise value:

$$1 \text{ lb.} = 454 \text{ g} \qquad \text{(EQ 2.6)}$$

Do we use 5 significant figures (453.59 g) or 3 significant figures (454 g)? We will answer this question when we discuss the guidelines for determining the correct number of significant figures during the course of performing mathematical operations. At this point, it may be useful to add these summary statements:

- The level of uncertainty is determined by the precision of our measurement.
- Precision refers to the reproducibility of a series of measurements.
- Significant figures are used to express the level of precision for a given measurement.
- Most of the numbers that we encounter are inexact numbers.

roble

Rounding Off Numbers in Multiplication and Division

When numbers are multiplied or divided, the same rule applies; the result cannot be more precise than the least precise number used in the calculation. The least precise number is the number with the fewest number of significant figures, regardless of the position of the decimal point. Rounding is treated in the same manner:

6.144 miles / 0.15 hours = 40.96 miles/hr. rounded to two significant figures to give 41 mi/hr.

13.6 g/mL x 45.68 mL = 621.248 g rounded to three significant figures to give 621 g

(24.7 x 345.24) / 12.441 = 685.4294671 rounded to 3 significant figures to give 685

Summary

- For addition and subtraction, the number of decimal places in the calculated result is equal to the number of decimal places in the least precise number.
- For multiplication and division, the number of significant figures in the calculated result is equal to the number of significant figures in the least precise number.

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